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Logical operations based on topological concepts

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Abstract

A topology is characterized on Cμ,ν(Y, Z), the assortment of continuous mappings between two generalized topologies on Y and Z. Using continuous convergence of g-nets, acceptability and it are described to part Ness. The (μ, ν)- topology on Cμ,ν(Y, Z) is viewed as acceptable. Point-open topology and reduced open topology are characterized and investigated for generalized topological spaces.

Keywords: Function space; Generalized topology; g-Net; Continuous convergence.

Introduction

Investigations into the different parts of the function spaces between topological spaces has been a functioning area of examination in topology. A few examination papers have come up as of late dealing with various qualities of these spaces as well as highlighting a few immaculate issues. Nonetheless, in spite of this, one perspective actually stays unanswered in this regard. What occurs, on the off chance that Y and Z are furnished with generalized topological designs? The issue has become more relevant considering the way that in numerous areas of examination, where topological methodology is being applied, we go over structures which don't shape a topology, yet is a generalized type of topology.

Preliminaries

Definition 2.1. Let X be a non-empty set. A collection G of subsets of X is called a generalized topology (GT, in brief) on X if

(i) $\emptyset \in \mathcal{G}$,

(ii) $\mathcal{G}$ is closed under arbitrary union.

The members of G are called generalized open sets (g-open sets, in brief), their complements are called generalized closed sets (g-closed sets, in brief).

Definition 2.2. Let (X, μ) and (Y, ν) be two GT S′ s. Then a map f: X → Y is said to be (μ, ν)-continuous if for U ∈ ν implies f^{-1} (U) ∈ μ, for every U ∈ ν.

It may be shown that f is continuous if and only if it is continuous for each x ∈ X, that is, for each neighborhood V of f(x), there exists a neighborhood U of x such that f(U) ⊆ V. The proof of the same is provided at the end of this section.

Now onward we write (μ, ν)-continuity as simply continuity unless there is any ambiguity.

In comparison of nets for general topology, generalized nets (g-nets, in brief) are developed for generalized topological spaces.

Definition 2.3.

A pre-ordered set is a pair (D, ≥) where D is a non-empty set and ≥ is binary relation in D which is reflexive and transitive.

Let X be a non-empty set and (D, ≥) be a pre-ordered set. A mapping s: D → X is called generalized net (g-net, in brief) on X. For n ∈ D, s(n) is denoted by s_n.

Convergence and sub net of a g-net are defined the way it is done for a net in topology. Below we provide the definitions of regular points and saddle points for generalized topological spaces.

Definition 2.4. Let (X, μ) be a generalized topological space. Then a point x ∈ X is called a saddle point if there does not exist any open neighborhood U of x. A point is called a regular point if it is not a saddle point.

Now we provide the proof of our proposed result about continuity:

Proposition 2.5. Let (X, μ) and (Y, ν) be two GT S′ s and f : X → Y be any mapping. Then the following are equivalent:

- f is continuous;
- inverse image of all generalized closed sets is closed;
- for every regular point x and any neighborhood V of f(x), there exists a neighborhood U of x such that f(U) ⊆ V;
- for each g-net {sn}n∈D converging to x, the image g-net {f(sn)}n∈D converges to f(x).
- f is continuous at each x ∈ X, that is, for each neighborhood V of f(x), there exists a neighborhood U of x such that f(U) ⊆ V.

Proof. For equivalence of $(i), (ii), (iii)$ and (iv), please refer to [13].
$(i) \Rightarrow (v)$: Let f be continuous and $x \in X$ be a regular point, then the result holds in view of (iii). Now, let $x \in X$ be not a regular point and V be any open neighbourhood of $f(x)$, that is $f(x) \in V$. Therefore $x \in f^{-1}(V)$. Since f is continuous and V is an open set in Y, therefore $f^{-1}(V)$ is an open set in X containing x, which leads to a contradiction as x is not a regular point. Therefore, if f is a continuous map from X to Y, then for every saddle point $x \in X$, there does not exist any open neighbourhood V of $f(x)$ and hence the result holds vacuously.
$(v) \Rightarrow (i)$: Let $V \subseteq Y$ be an open set in Y. We have to show that $f^{-1}(V)$ is an open set in X. If $f^{-1}(V) = \emptyset$, we are done. Therefore, we consider $x \in f^{-1}(V)$, then $f(x) \in V$. Hence by the given hypothesis, there exists an open neighbourhood U of x such that $f(U) \subseteq V$. Thus $x \in U \subseteq f^{-1}(V)$, that is, $f^{-1}(V)$ is an open set in X. This completes the proof. □

A topology on $C_{\mu,\nu}(Y, Z)$

Let (Y, μ) and (Z, ν) be two GT S′ s. For U ∈ μ, V ∈ ν and any non-empty subset F of Z^Y, we define

$$(U, V) = \{f \in F \subseteq Z^Y \mid f(U) \subseteq V\},$$

where Z^Y is the collection of all the functions from (Y, μ) to (Z, ν) and F ⊆ Z^Y. Let $S_{\mu,\nu} = \{(U, V) \mid U \in \mu, V \in \nu\}$.

Definition 3.1. $S_{\mu,\nu}$ is a sub basis for a topology on F.

Proof. For f ∈ F, we have, f(∅) ⊆ V , for each V ∈ ν. As ∅ ∈ μ, we get f ∈ (∅, V) for each V ∈ μ. Therefore ∪ $S_{\mu,\nu}$ = F.

Definition 3.2. Let (Y, μ), (Z, ν) be two $GTS's$. Let (X, λ) be another GTS. For a function $g : X \times Y \to Z$, we can define a mapping $g^* : X \to \mathcal{C}_{\mu,\nu}(Y, Z)$ by $g^*(x)(y) = g(x, y)$.

The mappings g and g^* related in this way are called *associated maps*.

Definition 3.3. Let (Y, μ) and (Z, ν) be two GTS's. A topology τ on $\mathcal{C}_{\mu,\nu}(Y, Z)$ is called

(i) *admissible* if the evaluation mapping $e : \mathcal{C}_{\mu,\nu}(Y, Z) \times Y \to Z$ defined by $e(f, y) = f(y)$ is continuous.

(ii) *splitting* if for each GTS X, continuity of $g : X \times Y \to Z$ implies the continuity of $g^* : X \to \mathcal{C}_{\mu,\nu}(Y, Z)$, where g^* is the associated map of g.

In the next theorem, a characterization of admissibility is provided for $GTS's$.

Theorem 3.4. Let (Y, μ), (Z, ν) be two GTS's. A topology τ on $\mathcal{C}_{\mu,\nu}(Y, Z)$ is admissible if and only if for each GTS X, continuity of $g^* : X \to \mathcal{C}_{\mu,\nu}(Y, Z)$ implies continuity of $g : X \times Y \to Z$, where g is the associated mapping of g^*.

Proof. Let τ be admissible and $g^* : X \to \mathcal{C}_{\mu,\nu}(Y, Z)$ be continuous. We define $h : X \times Y \to \mathcal{C}_{\mu,\nu}(Y, Z) \times Y$ by $h(x, y) = (g^*(x), y)$. If e is the evaluation map, we have $e \circ h : X \times Y \to Z$, given by $(e \circ h)(x, y) = e(h(x, y)) = e(g^*(x), y) = g^*(x)(y) = g(x, y)$. As e and h are continuous, we have $g = e \circ h$ is continuous.

Conversely, let the condition hold. We take $X = \mathcal{C}_{\mu,\nu}(Y, Z)$. Then $g^* : X \to \mathcal{C}_{\mu,\nu}(Y, Z)$, where g^* is taken as the identity map, is continuous. Hence $g : \mathcal{C}_{\mu,\nu}(Y, Z) \times Y \to Z$ is continuous. Now, $g(f, y) = g^*(f)(y) = f(y)$. Thus g is nothing but the evaluation mapping which is continuous. □

Conclusion

The emergence of topology in the development of a few rough functions will be the bridge for some applications and will find the secret relations between information. Topological generalizations of the idea of rough functions open the way for connecting rough coherence with the area of close to continuous functions. Utilizations of topological rough functions of data systems open the entryway about the numerous changes among various kinds of data systems, for example, multivalued and single-esteemed data systems.

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